SIMULTANEOUS OSCILLATIONS AT THREE FREQUENCIES IN A REGENERATIVE CIRCUIT WITH A LIMITER TYPE NON-LINEAR ELEMENT*

B. N. BISWAS

DEPARTMENT OF PHYSICS, THE UNIVERSITY OF BURDWAN, WEST BENGAL.

(*Received August 19, 1964*)

ABSTRACT. In this paper the phenomenon of simultaneous oscillations at three anharmonically related frequencies in a regenerative loop, containing a limiter type non-linear element, has been analysed. The effects of finite selectivity of the modes on the locking range have been studied. The response of such a loop to an external input has also been analysed. A possible method for the elimination of the 'three-frequency effect' in such a circuit has been suggested. An experimental arrangement of such a regenerative loop, containing adjustable selectivity and gain control arrangement, has been described. Experimental results have also been presented in support of the conclusions of the analysis

INTRODUCTION

The weak-signal suppression effect in non-linear regenerative tuned circuits has been examined by many authors. It is known that compression type characteristics are mutually destructive and expander type characteristics, on the other hand, are mutually supporting.

In this paper it will be shown that the compression type characteristics of a limiter may sometimes become mutually supporting in nature with respect to signals having certain phase and amplitude relationship among themselves. In particular, if the input to the limiter consists of three components the frequencies of which are anharmonically related and the phases are related as in a phase modulated wave and further if the amplitudes of the different signals are properly related, then the stronger signals may help the growth of the weaker ones. Thus with respect to such singnals the destructive character of the limiter is lost. Therefore if the limiter is incorporated in a regenerative loop containing adjustable frequency selective networks and further if the gain of the loop for different modes is properly adjusted, then simultaneous oscillations at three anharmonically related frequencies can be maintained.

* This work has been done at the Institute of Radio Physics and Electronics, University of Calcutta, Calcutta.

This is a desirable result where continuously variable stable frequency oscillations are needed, e.g., in frequency synthesis. In some cases, however, it may cause serious trouble, e.g., in Automatic phase control circuits, where depending upon the gain and transmission characteristics of bandpass tuned circuits, the system may break into simultaneous oscillations at different frequencies.

In section 2 the transmission characteristics of two types of non-linear elements one having the limiter type non-linear transference and the other having the expandor type non-linear transference—in the presence of three signals having certain phase relationships among themselves, have been briefly studied. It has been pointed out that it is possible to realise simultaneous oscillations at three frequencies in a feedback circuit which can independently support three distinct modes and contains a limiter type non-linear element in the loop. The region of amplitude stability has also been found out theoretically.

In section 3 the effects of finite selectivity of the modes on the shifts of the frequencies of oscillations from the resonant frequencies of the tuned circuits have been studied. The possibility of having continuously variable stable frequency oscillations has also been suggested.

Section 4 deals with the effect of an external input, having a frequency nearly equal to that of any one of the free running modes. Expressions for the critical value of the amplitude of the external input for quenching action and the corresponding expression for locking range have been found out.

In section 5, a method has been discussed for converting the regenerative loop, sustaining simultaneous oscillations at three-frequency, into a degenerative one with respect to the undesired components with the help of a non-linear phase shifting network.

In section 6, experimental arrangement of the regenerative loop containing the limiter type non-linear element has been described and experimental data with respect to the region of stable oscillations have also been presented, which are in good agreement with the region of amplitude stability found in Section 2.

TRANSMISSION CHARACTERISTICS

"Three frequency effect" in a non-linear element or Internal synchronisation

The phenomenon that gives rise to the loss of destructive character of a limiter with respect to three signals which have certain phase and amplitude relationship among themselves, is called the "three frequency effect". It is well known that when two non-coherent signals are applied to a limiter type non-linear transference, the strong signal will be captured and the weak signal will be rejected. The expandor type characteristics, on the other hand, helps the weaker signal to build up. But if the phases of the components of the signal to the limiter are related as in a phase modulated wave and further if the amplitudes are properly related, then the compressor type characteristic of the limiter type non-linear element will fail to be mutually destructive in nature, i.e. the presence of the stronger signals will help the growth of the weaker one. In this section this phenomenon will be briefly studied with respect to two types of non-linear elements one having the limiter type non-linear transference and the other having the expansion type non-linear transference.

Let us consider that the inputs to the non-linear elements are

$$e_A = A\cos(w_A t+\phi_1) = A\cos\psi_A,\ e_B = B\cos(\omega_B t+\phi_2) = B\cos\psi_B,$$
$$e_C = C\cos(\omega_C t+\phi_3) = C\cos\psi_C, \qquad \dots \quad (2.1a)$$

where A, B, C are the peak amplitudes of the signals and ψ_A, ψ_B and ψ_C are their instantaneous phases which are related as

$$\psi_A+\psi_C = 2\psi_B+\phi. \qquad \dots \quad (2.1b)$$

The input output characteristic of the non-linear element having the compressor type characteristics is assumed to be given by

$$X_{out} = a_1 X_{in} - a_3 X_{in}^3, \qquad \dots \quad (2.2)$$

and that of the expandor type non-linear element is of the form

$$X_{out} = \sinh(X_{in}), \qquad \dots \quad (2.3)$$

where 'X_{in}' and 'X_{out}' are respectively the input and output of the non-linear element.

Therefore the outputs of the expandor type non-linear element consisting of frequencies $\omega_A = d\psi_A/dt$, $\omega_B = d\psi_B/dt$ and $\omega_C = d\psi_C/dt$ are given respectively by

$$T_A = 2[I_0(B)I_0(C)I_1(A)+I_0(A)I_2(B)I_1(C)\cos\phi], \qquad \dots \quad (2.3a)$$
$$T_B = 2[I_0(A)I_0(C)I_1(B)+I_1(A)I_1(B)I_1(C)\cos\phi], \qquad \dots \quad (2.3b)$$
$$T_C = 2[I_0(A)I_0(B)I_1(C)+I_0(C)I_1(A)I_2(B)\cos\phi], \qquad \dots \quad (2.3c)$$

where $I_k(Z)$ is modified Bessel Function of order 'K' and argument 'Z'. Taking $\phi = (2n+1)\pi$ for the steady state, Eqs. (2.3a) (2.3b) and (2.3c) reduce to

$$T_A = 2[I_0(B)I_0(C)I_1(A)-I_0(A)I_2(B)I_1(C)], \qquad \dots \quad (2.3d)$$
$$T_B = 2[I_0(A)I_0(C)I_1(B)-I_1(A)I_1(B)I_1(C)], \qquad \dots \quad (2.3e)$$
$$T_C = 2[I_0(A)I_0(B)I_1(C)-I_0(C)I_2(B)I_1(A). \qquad \dots \quad (2.3f)$$

The plots of T_A, T_B and T_C are shown in Fig. 1 for the case when $A = C$ for different values of B. If the input to the expandor type non-linear element consists of three components e_A, e_B and e_C, and further if one assumes that the amplitude of e_A is equal to that of e_C and the amplitude of e_B is less than that of either e_A

or e_C then one can find from the plots of Fig. 1 that after sometime all the signals will be found to grow, particularly the weaker one. Thus expandor type characteristics help building up of weaker ones.

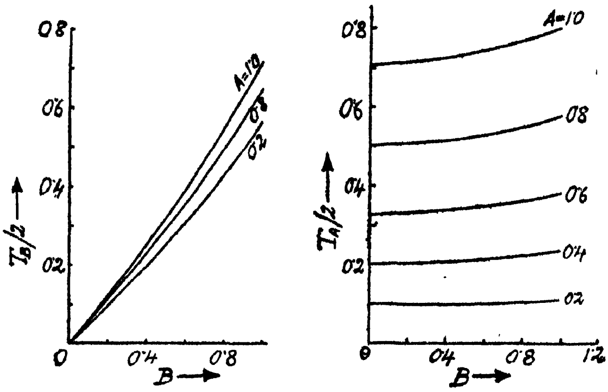


Fig. 1(a) & 1(b): The input-output characteristics for the three different components of a composite signal of an expandor type non-linear element. The components of the composite signal bear a definite phase relationship among themselves, namely $\psi_A+\psi_C = 2\psi_B+\pi$

Let us now consider the limiter type characteristic as represented by Eq. (2.2).

The outputs of the limiter type non-linear element consisting of frequencies $\omega_A = d\psi_A/dt$, $\omega_B = d\psi_B/dt$ and $\omega_C = d\psi_C/dt$ are respectively given by

$$T_A = [A-\{A^3+2(B^2+C^2)A+B^2C\cos\phi\}], \tag{2.4a}$$

$$T_B = [B-\{B^3+2(A^2+C^2)B+2ABC\cos\phi\}], \tag{2.4b}$$

$$T_C = [C-\{C^3+2(B^2+A^2)C+AB^2\cos\phi\}]. \tag{2.4c}$$

When the signals are non-coherent the $\cos\phi$ terms drop out and the corresponding outputs are given by

$$T_A = [A-\{A^3+2(B^2+C^2)A\}], \quad \ldots \tag{2.5a}$$

$$T_B = [B-\{B^3+2(A^2+C^2)B\}], \quad \ldots \tag{2.5b}$$

$$T_C = [C-\{C^3+2(A^2+B^2)C\}]. \quad \ldots \tag{2.5c}$$

Now for the case when the phases are related as in Eq. (2.1b) taking $\phi = (2n+1)\pi$ for the steady state, we have the following expressions for the output of the limiter type non-linear element for the different frequencies:

$$T_A = [A-\{A^3+2(B^2+C^2)A-B^2C\}], \quad \ldots \tag{2.4d}$$

$$T_B = [B-\{B^3+2(A^2+C^2)B-2ABC\}], \quad \ldots \tag{2.4e}$$

$$T_C = [C-\{C^3+2(A^2+B^2)C-AB^2\}]. \quad \ldots \tag{2.4f}$$

The plots of T_A, T_B and T_C given by Eqs. (2.4d), (2.4e) and (2.4f) are shown in Fig. 2. It is seen from the plot for the case where $A = C$, that when this type

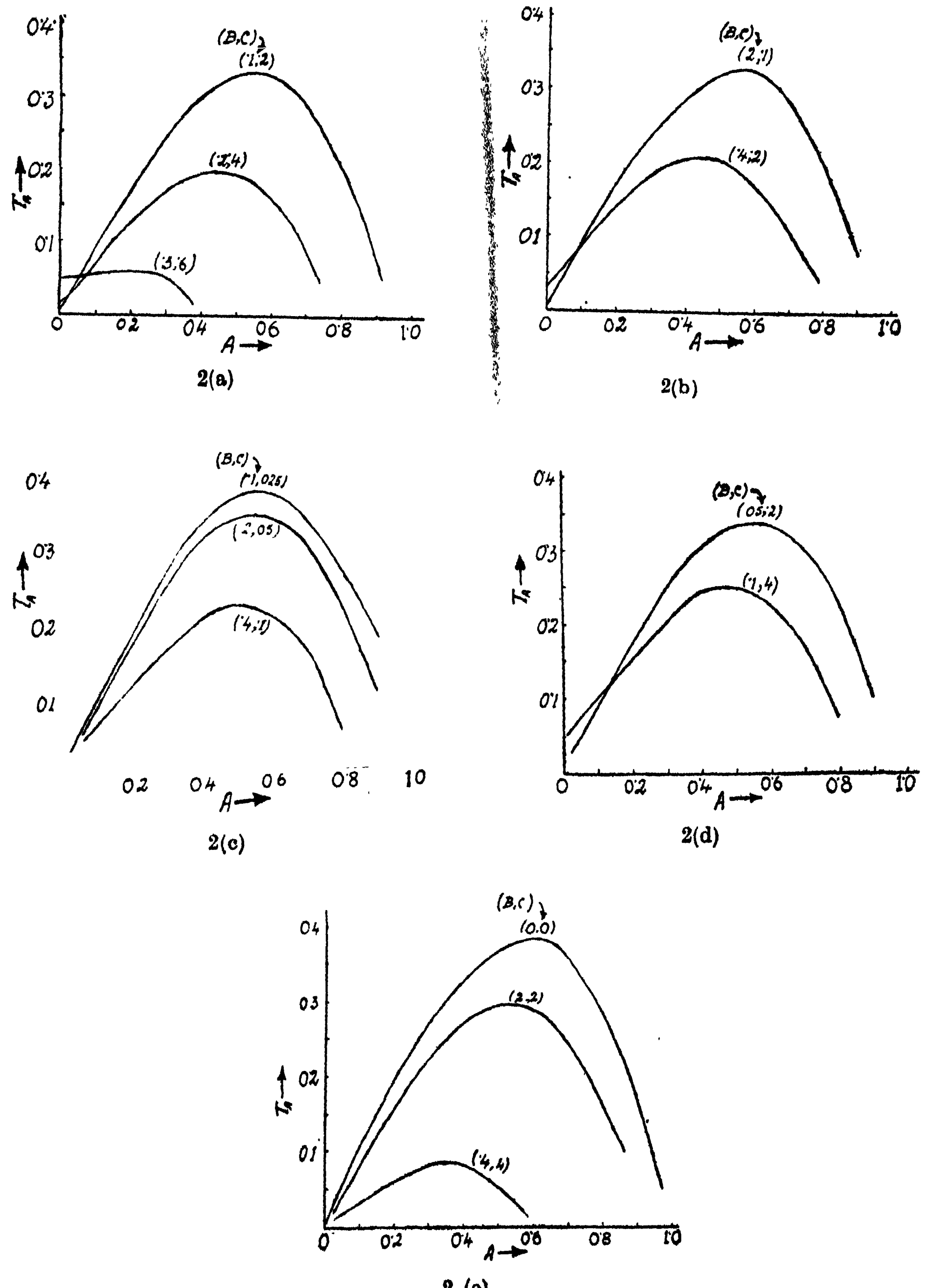
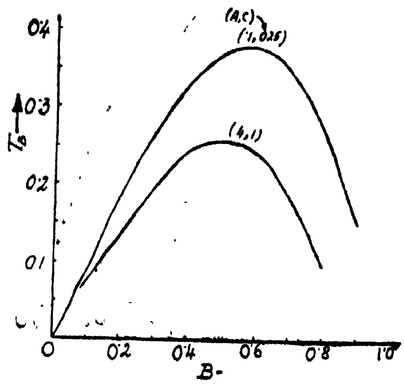
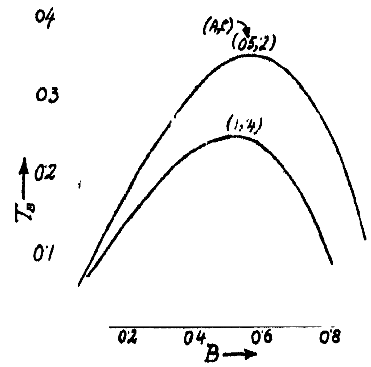


Fig. 2(a)–2(e) : Transfer characteristic of a limiter for the component 'A' of a composite signal consisting of three components that bear the phase relationship $(\psi_A + \psi_C = 2\psi_B + \pi)$ among themselves.

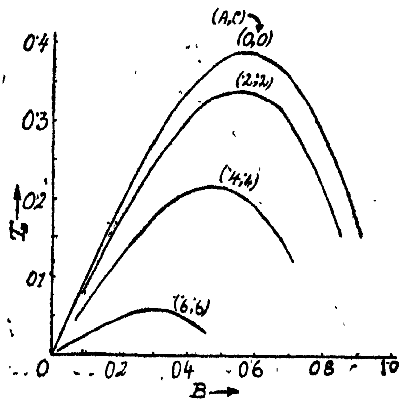
of limiter type non-linear element is incorporated in a regenerative loop and when B is greater than A or C, it does not help the growth of either A or C. For example if $B = 0.8$ and $A = 0.1$ then it is seen that A becomes non-existent after



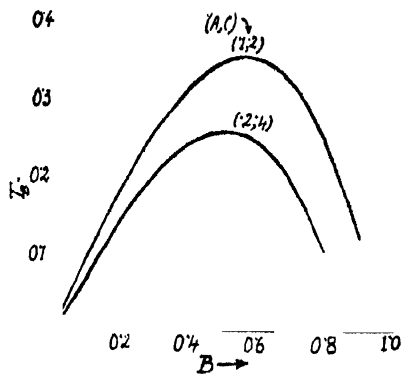
2 (f)



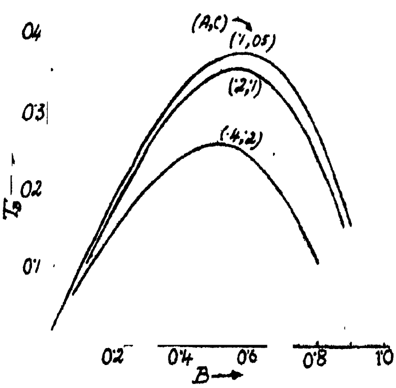
2 (g)



2 (h)



2 (i)



2 (j)

Fig 2(f)–2(j): Transfer characteristic of a limiter for the component 'B' of a composite signal consisting of three components that bear the phase relationship ($\psi_A + \psi_C = 2\psi_B + \pi$) among themselves sometime. But when A or C is greater than B then it is seen from the plots that A helps the growth of B. But it is to be noted that when A is much greater than B then the loop acts in such a way as to help the elementation of B. Thus there are limits to the value of the ratio A/B for which the loop may act in such way as to help building up of the weaker one. From the above discussion it is clear that if the non-linear element is placed in a regenerative loop having certain gain-frequency relations, the loop may break into simultaneous oscillations at the three frequencies ω_A, ω_B and ω_C. A schematic diagram of a circuit for producing simultaneous oscillations at three frequencies is shown in Fig. (3), which contains three gain control arrangements for the oscillations at frequencies ω_A, ω_B and ω_C respectively and three frequency selective networks for the oscillating components A, B, and C respectively.

Region of Stability of Amplitudes

From the above discussion it is evident that simultaneous oscillations at three frequencies will be maintained in the regenerative loop containing the limiter type non-linear element if the gain, phase and transmission characteristics of the loop for a particular component bear certain relationships with those of the others. Further the presence of any one of the components will have pronounced effect on the gain, phase and the transmission characteristics of the others. Hence if the amplitude of oscillation of any one of the components is changed, the corresponding transmission characteristics of the other will be modified, as a result of which all the modes may exist with a modified amplitude distribution. The above situation will occur if the gains of the loop for different components bear certain relationship among themselves. In this selection this region of amplitude stability has been found out.

Let us consider the regenerative loop shown in Fig. 3. It contains gain control arrangements G_A, G_B and G_C respectively for the components e_A, e_B and e_C and

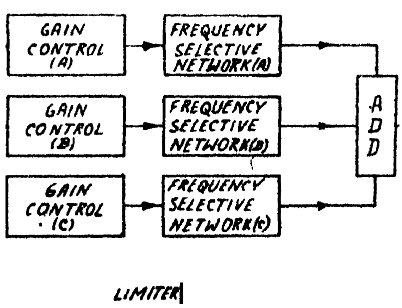


Fig 3 : Schematic diagram of a regenerative loop with the limiter for simultaneous oscillations at three anharmonically related frequencies.

a limiter type non-linear element. The input-output relation of the compressor type non-linear element is assumed to be given by

$$X_{out} = a_1(X_{in}) - a_3(X_{in})^3, \quad \dots \quad (2.5)$$

where

$$X_{in} = A\cos\psi_A + B\cos\psi_B + C\cos\psi_C. \quad \dots \quad (2.6)$$

Then the instantaneous amplitude and phase equations can be written as

$$(1-a_1G_A)A + T\frac{dA}{dt} = -\frac{3}{4}a_3G_A[A^3+2(B^2+C^2)A+B^2C\cos\phi], \quad \dots \quad (2.7)$$

$$(1-a_1G_B)B + T\frac{dB}{dt} = -\frac{3}{4}a_3G_B[B^3+2(A^2+C^2)B+2ABC\cos\phi], \quad (2.8)$$

$$(1-a_1G_C)C + T\frac{dC}{dt} = -\frac{3}{4}a_3G_C[C^3+2(A^2+B^2)C+AB^2\cos\phi], \quad (2.9)$$

and

$$T\frac{d\phi}{dt} = 3a_3\left[G_BAC + \frac{1}{4}\left(G_A\frac{A}{C} + G_C\frac{C}{A}\right)\right]\sin\phi, \quad \dots \quad (2.10)$$

where T is an appropriate constant.

Putting

$$\frac{4}{3}\cdot\frac{a_1G_A-1}{a_3G_A} = K_A$$

$$\frac{4}{3}\cdot\frac{a_1G_B-1}{a_3G_B} = K_B \quad \dots \quad (2.11)$$

$$\frac{4}{3}\cdot\frac{a_1G_C-1}{a_3G_C} = K_C$$

and $\phi = (2n+1)\pi$, we have in the steady state

$$3A^3 - 6A^2C + (4C^2+K_A-2K_B)A + (K_B-2C^2)C = 0, \quad \dots \quad (2.12)$$

and

$$3C^3 - 6AC^2 + (4A^2+K_C-2K_B)C + (K_B-2A^2)A = 0. \quad \dots \quad (2.13)$$

Let us assume that the frequencies of the components A and C are symmetrically situated with respect to the frequency of oscillation of the component B. In general 'A' is not equal to 'C' and so let us assume that

$$C = mA \quad \dots \quad (2.14)$$

where 'm' is a positive number. Substituting the value of C from (2.14), in the steady state we have

$$B^2 = \frac{K_A-(1+2m^2)A^2}{2-m}, \quad \dots \quad (2.15a)$$

$$= \frac{K_C-(m^2+2)A^2}{2-1/m} \qquad \dots \quad (2.15b)$$

and

$$A^2 = \frac{K_A-(2-m)K_B}{2m^3-4m^2+6m-3}, \qquad \dots \quad (2.16a)$$

$$= \frac{mK_C-(2m-1)K_B}{2-4m+6m^2-3m^3} \qquad \dots \quad (2.16b)$$

The above equations have been plotted in Fig. 4 where the region of stable oscillation which lies within the bounding curves corresponding to $A^2 = 0$ and

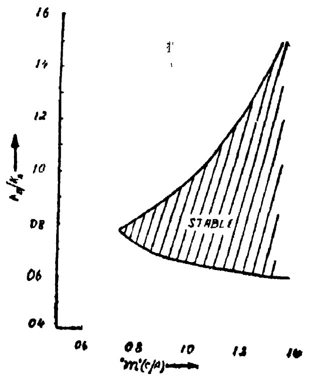


Fig 4: Amplitude stability diagram showing the amplitude relationship among the three components for simultaneous oscillations at three anharmonically related frequencies

$B^2 = 0$. Now if the gains are symmetrically distributed around the frequency of oscillation of 'B' i.e. $K_A = K_C$ and $A = C$, we have

(2.17)

$$B = \sqrt{2K_B-2K_A}. \qquad (2.18)$$

Therefore for simultaneous oscillations at three frequencies in the symmetrical case we must have

$$K_A = K_C > K_B, \qquad (2.19)$$

and

$$3K_B > 2K_A. \qquad (2.20)$$

In the above discussion nothing has been said about the stability of the amplitudes of oscillations. To study the question of stability of the modes we shall follow the Liapunoff technique. We imagine that the amplitudes 'A', 'B' and 'C' are given respectively increments of 'x', 'y' and 'z' about their mean values of the amplitude of oscillations i.e. A_0, B_0 and C_0. If it is found that these increments ultimately die out we call the oscillation modes stable and otherwise unstable. Thus we have the incremental equations for the three components from

$$T\frac{\partial x}{\partial t} = -\frac{3}{4}a_3G_A\{(5A_0^2+2B_0^2-K_A)x+2A_0B_0y+(4A_0^2-B_0^2)z\}, \quad \dots \quad (2.21)$$

$$T\frac{\partial y}{\partial t} = -\frac{3}{4}a_3G_B\{(3B_0^2+4A_0^2-K_B)y+2A_0B_0x+2A_0B_0z\}, \quad \dots \quad (2.22)$$

$$T\frac{\partial z}{\partial t} = -\frac{3}{4}a_3G_C\{(5A_0^2+2B_0^2-K_C)z+(4A_0^2-B_0^2)x+2A_0B_0y\}. \quad \dots \quad (2.23)$$

Putting $p = d/\partial t$ the characteristic equation can be written, for the simple case when $A = C$, i.e. $K_A = K_C$, as

$$p^2+\frac{3}{4}a_3T[G_B(2K_B-K_A)+6G_A(K_A-K_B)]p$$

$$+\left(\frac{3}{4}a_3\right)^2 G_AG_B[2(K_A-K_B)^2+4(K_A-K_B)(3K_B-2K_A)] = 0 \quad \dots \quad (2.24)$$

From the above Eq. (2.24) it is seen that the coefficients are real and positive when the conditions of Eqs. (2.19) and (2.20) are satisfied. Therefore Eqs. (2.19) and (2.20) give the correct criteria of simultaneous oscillations at three frequencies in the loop.

EFFECT OF TUNED CIRCUITS ON THE LOCKING RANGE

In the above analysis we have assumed that all the three oscillations occur at the resonant frequencies of the three tuned circuits. If only one mode exists the oscillation does occur at the resonant frequency of the corresponding tuned circuits. If, however, the three modes are present simultaneously it is possible that the frequencies of oscillations will differ from the resonant frequencies. Now from the analysis in section 2, it is clear that frequency and phase condition in Eq. (2.1b) must be satisfied. It is also evident that the amount of detuning present in the different tuned circuits will have a marked effect on the overall characteristics. This effect is studied in this section.

Let us suppose that the three tuned circuits which have been incorporated in the loop have sufficiently high Q-values and the three frequencies of oscillations that are simultaneously present in it, are chosen in such a way that the gain of any of the tuned circuits at its centre frequency is considerably large compared to that at other two frequencies which are away from the centre frequency.

Considering the loop shown in Fig. 3, one finds that for the simultaneous oscillations to occur at the three frequencies, the linear and non-linear gains of the loop for the three frequencies are to be related at

$$A = G_A(p)T_A(A, B, C), \qquad \dots \quad (3.1)$$

$$B = G_B(p)T_B(A, B, C), \qquad \dots \quad (3.2)$$

$$C = G_C(p)T_C(A, B, C), \qquad \dots \quad (3.3)$$

where $G_A(p)$, $G_B(p)$ and $G_C(p)$ are the linear gains of the loop for the component A, B and C respectively $\frac{1}{A}\ T_A(A, B, C)$, $\frac{1}{B}\ T_B(A, B, C)$ and $\frac{1}{C}\ T_C(A, B, C)$ are the corresponding non-linear gains of the loop. From Eqs. (3.1), (3.2) and (3.3) one can write the corresponding amplitude and phase equations as (see Appendix A)

$$\frac{2}{\alpha_A} \cdot \frac{dA}{dt} \simeq GA_0 \left[a_1 A - \frac{3}{4}\ a_3\{A^3 + 2(B^2+C^2)A + B^2C \cos\phi\} \right] - A, \qquad \dots \quad (3.4)$$

$$\frac{2}{\alpha_B} \quad \frac{dB}{dt} \quad GB_0 \left[a_1 B - \frac{3}{4}\ a_3\{B^3 + 2(A^2+C^2)B + 2ABC \cos\phi\} \right] - B, \qquad \dots \quad (3.5)$$

$$\frac{2}{\alpha_C} \cdot \frac{dC}{dt} \simeq GC_0 \left[a_1 C - \frac{3}{4}\ a_3\{C^3 + 2(A^2+B^2)C + AB^2 \cos\phi\} \right] - C, \qquad \dots \quad (3.6)$$

and

$$\frac{d\phi}{dt} \simeq 2\Delta\omega_B - (\Delta\omega_A + \Delta\omega_C) + K \sin\phi, \qquad \dots \quad (3.7)$$

where

$$K \simeq \frac{3}{8} \cdot a_3 \left[\alpha_A GA_0 \frac{C}{A} + \alpha_C GC_0 \frac{A}{C} + 2\alpha_B GB_0 AC \right], \qquad \dots \quad (3.8)$$

and $\Delta\omega_A$, $\Delta\omega_B$ and $\Delta\omega_C$ are respectively the detuning from the centre frequencies of components A, B and C. α_A and α_B and α_C are proportional to the quality factors of the three tuned circuits, viz.,

$$\alpha_A = \frac{\omega_{0A}}{Q_A},\ \alpha_B = \frac{\omega_{0B}}{Q_B} \text{ and } \alpha_C = \frac{\omega_{0C}}{Q_C}.$$

In the steady state $d\phi/dt = 0$ and the equilibrium phase ϕ is given by

$$K \sin\phi = 2\Delta\omega_B - (\Delta\omega_A + \Delta\omega_C), \qquad \dots \quad (3.9)$$

or,

$$\frac{2\Delta\omega_B - (\Delta\omega_A + \Delta\omega_C)}{K} \leqslant 1. \qquad \dots \quad (3.9a)$$

Obviously for entrainment the value of K must be greater than $|2\Delta\omega_B-(\Delta\omega_A+\Delta\omega_C)|$

This defines the limits of detuning permissible. Beyond this limit the phase relations change to a value that the feedback circuit acts in a way so as to capture the strong signal and to reject the small signal.

The amplitude equations are given by

$$KA_o = A^2+2(B^2+C^2)+B^2\frac{C}{A}\cos\phi, \qquad \dots \quad (3.10)$$

$$KB_o = B^2+2(A^2+C^2)+2AC\cos\phi, \qquad \dots \quad (3.11)$$

$$KB_o = C^2+2(A^2+B^2)+B^2\frac{A}{C}\cos\phi. \qquad \dots \quad (3.12)$$

In general A is not equal to C taking $C = mA$ where 'm' is a positive number (integer or fraction) one can write from Eqs (3.10), (3.11) and (3.12) the following relations.

$$B^2 = \frac{KA_o-(1+2m^2)A^2}{2+m\cos\phi}, \qquad (3.13a)$$

$$= \frac{K_{C0}-(2+m^2)A^2}{2+1/m\cos\phi}, \qquad (3.13b)$$

$$= KB_o-\{2(1+m^2)-2m\cos\phi\}A^2, \qquad (3.13c)$$

and

$$A^2 = \frac{KA_o-(2+m\cos\phi)KB_o}{(1+2m^2)-2(1+m^2+\cos\phi.m)(2+m\cos\phi)}, \qquad \dots \quad (3.14a)$$

$$= K_{C0}-KA_o-\left(\frac{1}{m}-m\right)B^2\cos\phi, \qquad \dots \quad (3.14b)$$

and therefore the limiting value of the equilibrium phase ϕ is given by

$$\text{Cos}\,\phi \simeq 5\,\frac{KA_o-KB_o}{KB_o-2KA_o} \qquad . \quad (3.15)$$

From a study of the above equations it is clear that if the tuned circuits have finite selectivity, then simultaneous oscillations at three frequencies may occur in the regenerative loop at frequencies which are not necessarily the resonant frequencies of the respective tuned circuits. In such a case the amounts of detuning from the resonant frequencies will be automatically adjusted by the loop in accordance with the amplitudes of the different modes, the selectivity o fthe tuned circuits and the type of non-linear characteristic used. Ordinarily due to power supply variation, temperature fluctuations etc., the frequency of oscillation of the different modes will try to drift. But in this case the tendency will be influenced by the mutual coupling between the modes (vide Eq. 3.9).

We have so far considered the case when all the tuned circuits have more or less the same amount of selectivity. Let us now consider the case when one of the tuned circuits has a very high selectivity. For example, let us take case when the selectivity of the circuit sustaining the mode B is high, then the Eq. (3.7) reduces to

$$\frac{d\phi}{dt} \simeq -(\Delta\omega_A + \Delta\omega_C) + K \sin\phi. \qquad \dots \quad (3.16)$$

In the steady state it is seen from Eqs. (3.8) and (3.16) that the sum of the detunings of the mode A and the mode C from their respective centre frequencies is approximately constant. This means that if either the mode A or the mode C tries to drift in frequency then the other mode will drift in frequency in the opposite sense. This phenomenon gives an amount of stability to frequencies of simultaneous oscillations at three frequencies in regenerative loop containing the limiter type non-linear element.

EFFECT OF AN EXTERNAL INPUT

It is obvious that the first effect of the application of the external input to the system will be to cause an output at the frequency of excitation to appear. The amplitude of this output will depend upon the relative amplitudes of the free-running modes and the exciting signal at the input to the non-linearity and the linear response of the system at the exciting frequency. It is evident, for example, that if the exciting signal has a very large amplitude it may cause suppression of the internal modes and the output will then depend entirely on the input strength.

In general the effect of the external input will be to cause a reduction of the non-linear gains of the system. If the frequency of the external signal is close to that of any of the internal modes, there may also be an amount of energy exchange which may result in synchronisation of that internal mode with the applied signal. It can be readily shown that the effective gain of the relevant mode when it is locked to the external signal is increased by a factor that depends on the output and the phase difference between the two. The amplitude and phase equations of section 3 will have to be modified to take into account these effects, viz., reduction of the non-linear gains, the energy exchange and consequent frequency pulling. The relevant equations are presented in appendix B.

Although it is possible to solve these equations and treat the problem in all its generality it is considered advisable, for reasons of simplicity, to analyse only the following simple cases.

In the first case we assume that the loops for two of the modes are first disconnected and that for the other whose frequency is close to that of the exciting signal is closed and further that the amplitude and frequency of the external signal are such as to cause synchronisation. The loops for other two modes are subsequently closed after lock has been attained. Obviously we have to consider the phenomenon of single frequency synchronisation. The frequency of the free-running mode (say, B) will be pulled into synchronism with the external signal and consequently the amplitude of the relevant mode will change to B', given by

$$B' = B+E\cos\theta \tag{4.1}$$

where B is the amplitude of the free-running mode B in absence of the external input and E is the amplitude of the external signal at the input to the non-linearity and ϕ is the steady state phase difference between the external input and the free-running mode and is given by

$$\sin\,\theta = \Omega\,.\,\frac{E.2\omega_0}{B.\,Q_B} \tag{4.2}$$

where . $\Omega/2\pi$ is the difference of frequency between the external input and the free-running mode B and Q_B is quality factor of the tuned circuit sustaining the mode B. In this simple case the analysis of section 3 can be applied. It is to be remembered that B is to be changed to B'. Thus the amplitude of the external input necessary for quenching of oscillations at other two frequencies can be found out from the analysis of section 3.

In the second case we assume that the external input is applied to the system after internal synchronisation has been attained. Further it will be assumed that the amplitude and frequency relations of the external input with respect to any of the free running modes are such that it does not lock with any one of them. In this case also the external input will change the non-linear gains of the regenerative loop (in a way shown in appendix B). As a result the amplitude relations among the free-running modes will be altered in order that all the modes can be excited. Thus it follows from the discussion of section 3 that the frequencies of oscillations of all the free-running modes will be pulled to different values as permitted by the bandwidth of the tuned circuits. Analytical expressions for the modified amplitudes of the free-running modes and the different amount of detunings for the different modes can be found out from the analysis of section 3. It is to be remembered that Kz_0 of section 3 should be replaced by (Kz_0-2E^2) (see appendix B) where z stands for either A, B or C. Thus it is clear that if the exciting signal has such a large amplitude that internal synchronisation is lost then it will cause suppression of the internal modes and the output will depend entirely on the input strength and linear response of the system at the frequency of the external signal, If, however, the frequency of the external input is such that it lies at the centre of those of any two of the free-running modes and further if the linear response of the system at the frequency of the external input is adequate then there is a possibility of oscillations of the two internal modes in presence of the external signal.

ELIMINATION OF THREE FREQUENCY EFFECT

From the discussion of section 2 it is clear that a loop incorporating a limiter type non-linear element will help strong signals to build up and suppress the weaker ones. But if the net signal is a phase modulated one then the compressor type characteristics of the limiter will fail to be mutually destructive in nature. Thus the loop may still remain a regenerative one depending upon the amplitude and phase relationships among the various components. In this section a method has been discussed for converting the loop into a degenerative one with respect to the undesired components.

It has been shown in section 2 that simultaneous oscillations at three frequencies can be maintained if the gains at the three frequencies bear certain relationships depicted in Fig. 4 and the phase relations are such as in a phase modulated wave i.e.,

$$\psi_B - \psi_A = \psi_C - \psi_B. \tag{5.1}$$

This comes about because the suppression effect of the limiter is practically non-existent if the total instantaneous voltage has negligible amplitude variation with time and resembles a phase modulated wave. The above suggests a possibility of elimination of the weaker components by converting the phase modulated wave into an amplitude modulated one and amplitude limiting the latter. This conversion can be effected by means of a non-linear phase shifting network which

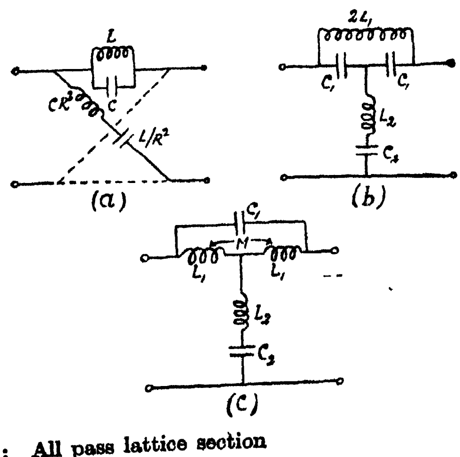


Fig 5(a): All pass lattice section

Fig 5(b): Bridged-T equivalent of Fig 5(a) for $\omega_1/K_1 > \sqrt{3}$

Fig 5(c): Bridged-T equivalent of Fig 5(a) for $\omega_1/K_1 < \sqrt{3}$

introduces a phase difference of 180° between the so-called side band components. It will be evident that other phase shifter circuits will have to be incorporated after the limiter in order to make the total phase shift at each of the frequencies equal to a multiple of 2π radians for the loop to be still regenerative.

The theory of the non-linear phase shifter is outlined below.

It is known that the total phase shift due to an allpass lattice composed of a parallel resonant circuit in the series arm and constant resistance inverse impedance in the shunt arm (Fig. 5a) is given by

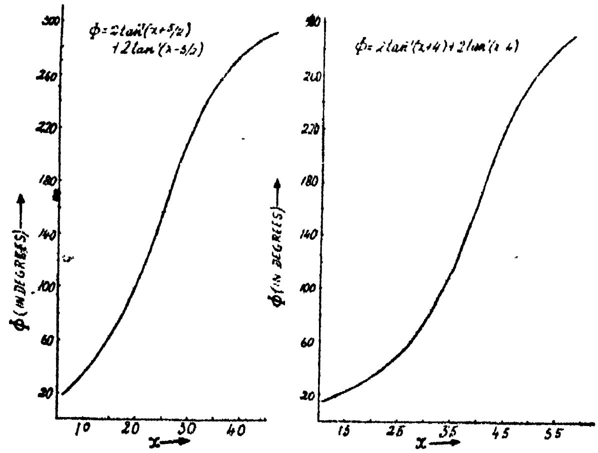
$$\phi = 2\left[\tan^{-1}\left(\frac{\omega+\omega_1}{K_1}\right) + \tan^{-1}\left(\frac{\omega-\omega_1}{K_1}\right)\right], \qquad \dots \quad (5.2)$$

where

$$2K_1 \quad , \frac{L}{R}, \qquad \dots \quad (5.3)$$

$$K_1^2+\omega_1^2 = \omega_0^2 .$$

To obtain an unbalanced equivalent of this network we apply the technique of conversion of lattice into a bridged T (see Fig. 5b). From the study of equation (5.1) it is evident that the nature of the phase shift depends on the parameter K. Fig. (6) shows the phase shift characteristics obtained with lattice or a bridged



(a) (b)

Fig 6: Phase-shift characteristics of the bridged-T network of Fig 5(a)

T having $\omega_1/K_1 = 5/2, 4$. It should be pointed out that when ω_1/K_1 is less than $\sqrt{3}$ the form shown in Fig. 5b cannot be used and one has to take recourse to the form shown in Fig. 5c. It should be mentioned that such phase shift characteristic can also be realised by active networks consisting of $R-C$ elements only.

Now putting $\omega/K_1 = x$ and $\omega_1/K_1 = q$ we have from (5.2)

$$\phi(x) = 2\tan^{-1}\left[\frac{2x}{1+q^2-x^2}\right]. \qquad \ldots \quad (5.4)$$

Now for converting a phase modulated wave to a corresponding amplitude modulated one we have the condition

$$\phi_A+\phi_C-2\phi_B = \pi, \qquad \ldots \quad (5.5)$$

where ϕ_A, ϕ_B and ϕ_C are the corresponding phase shifts for the components A, B and C respectively as they pass through the phase shifter and n is the number of stages utilised to obtain the required phase shift of π radian. Now comparing (5.3) and (5.4) we have

$$2\,\frac{x_A(1+q^2-x_C^2)+x_C(1+q^2-x_A^2)}{(1+q^2-x_A^2)(1+q^2-x_C^2)-4x_Ax_C} = -\frac{\tan(\pi/2n)+\tan(\phi_B)}{\tan(\pi/2n)\tan(\phi_B)-1}. \qquad \ldots \quad (5.6)$$

For $n = 2$, one can write from Eq. (5.6)

$$2\,\frac{x_A(1+q^2-x_C^2)+x_C(1+q^2-x_A^2)}{(1+q^2-x_A^2)(1+q^2-x_C^2)-4x_Ax_C} = \frac{1+\tan\phi_B}{1-\tan\phi_B}. \qquad \ldots \quad (5.7)$$

Therefore knowing the values of x_A, x_B and x_C the corresponding value of C can be found from Eq. (5.7).

EXPERIMENTAL SET-UP AND RESULTS

The experimental set up is shown in Fig. 7. The regenerative circuit consists of three variable gain and variable selectivity amplifiers and a limiter type non-

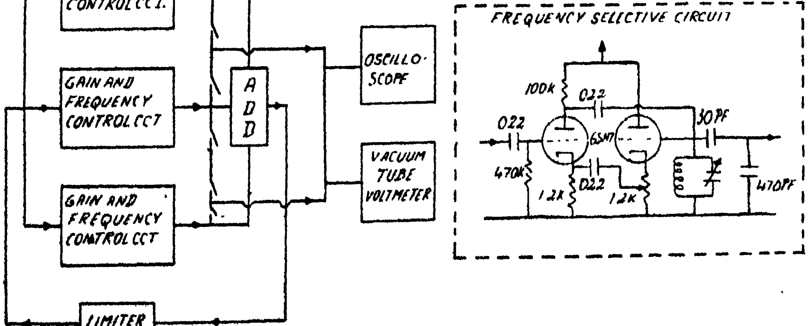


Fig 7: Experimental set-up The circuit diagram of the frequency selective network is also shown

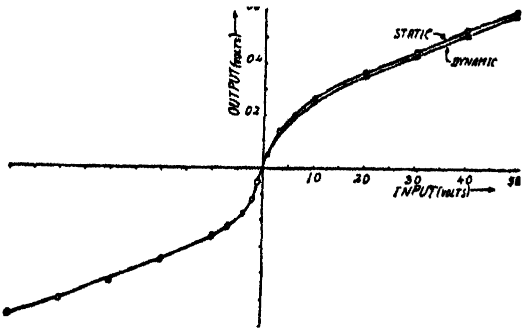


Fig 8: Experimentally obtained static and dynamic transfer characteristics of a limiter type non-linear element consisting of a pair of crystal diodes (1N34) connected back to back

linear element. The input-output characteristic of the limiter, consisting of a pair of crystal diodes (1N34) connected back to back, is shown in Fig. 8 and may be represented to a fair degree of approximation by

$$X_{out} = 0.42X_{in} - 0.16x_{in}^{3}, \quad x_{in} \leqslant 1$$

Now the gains for different modes were adjusted in such way as to cause the loop to break into simultaneous oscillations at three anharmonically related frequencies, viz. $f_A = 156$ Kc/s, $f_B = 184$ Kc/s and $f_C = 212$ Kc/s. The corresponding gains at frequencies f_A, f_B and f_C and their respective amplitudes were then measured. Thus knowing the values of the gains G_A, G_B and G_C and the values of the constant 'a_1' and 'a_3' the corresponding values of 'K_A' and 'K_B' were found out from Eq. (2.11). The ratios K_B/K_A and corresponding C/A have been plotted in Fig. 9. The equilibrium points, so found experimentally, lie within

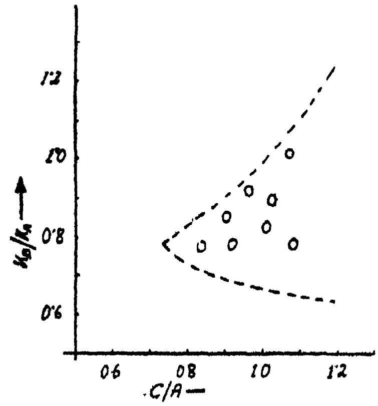
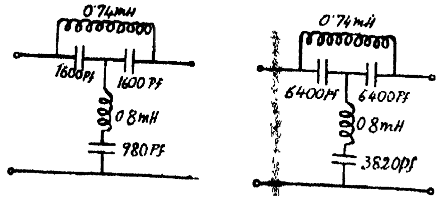


Fig 9: Amplitude stability diagram showing experimentally observed stable points. The computed bounding curve is shown dotted.

the region of amplitude stability found theoretically in section 2 and shown by the dotted line in the same figure for comparison.

The method of elimination of 'three frequency effect' in a regenerative loop containing a limiter type non-linear element, as discussed in section 5, requires a non-linear phase-shifting network N_1 (see Fig. 10a) for introducing a phase shift



(a) (b)

Fig 10: Shows one of the two identical sections of the bridged-T phase shifter

Fig. 10(a): $R=1.0K\,\Omega$, $\omega_{1,1}/K_1=4, \omega_{01}/K_1=\sqrt{17}, \omega_{01}=\sqrt{\frac{17}{9}}\,\omega_B, f_B=150Kc/s.$

Fig. 10(b): $R=500\,\Omega$, $\omega_{1,2}/K_2=4, \omega_{02}/K=\sqrt{17}, \omega_{02}=\sqrt{\frac{17}{36}}\,\omega_B, f_B=150Kc/s.$

of 180° between the so-called side-band components and it further requires a second non-linear phase shifting network N_2 (see Fig. 10b) to ensure that the net phase shifts suffered by the different modes in passing through the network are even multiples of π. The complete arrangement of the non-linear phase-shifting network with the limiter is shown in Fig. 11. To get rid of the 'three-frequency effect' in the regenerative loop, the non-linear phase shifting arrangement (Fig. 11) is to be introduced in the loop in the position shown dotted in Fig. 3.

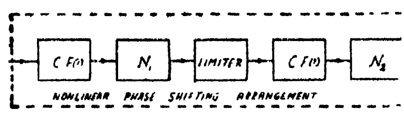


Fig 11: Arrangement of the non-linear phase shifting network with the limiter to be introduced for the elimination of the three-frequency-effect

As the components of the signals pass through this non-linear phase shifting network, they become incoherent in nature and the regenerative loop containing the limiter type non-linear element, will favour oscillation of that mode having the highest amplitude before the introduction of the non-linear phase shifter. The experiment performed fully confirmed the speculation.

CONCLUSION

Simultaneous oscillations at three anharmonically related frequencies in a regenerative loop containing a limiter type non-linear element have been analysed. The amplitude relations among the three modes for the co-existence have also been found out. Experimental results regarding their amplitude-relations have been presented and it has been found that the experimental results are in quite good agreement with those of the analysis. A possible method for the elimination of the three-frequency effect has also been suggested. The possibility of simultaneous oscillations at four and five frequencies in such a loop will be considered in a future communication.

ACKNOWLEDGMENT

The author takes the opportunity of thanking Mr. N. B. Chakrabarti of the Institute of Radio Physics and Electronics for suggesting the problem and for supervision of the work described. The author is indebted to Prof. J. N. Bhar, D.Sc., F.N.I., Head of the Institute of Radio Physics and Electronics for his kind interest and providing him with all the research facilities. The author also wishes to thank Prof. A. Mukherjee, D.Sc., Head of the Department of Physics, University of Burdwan, for his kind interest and encouragement.

APPENDIX

A.I. *Derivation of the loop equations for the case when the tuned circuits have finite Q-values* :

Let us consider the loop as shown in Fig. 3. The loop equations for different modes can be written as

$$A = G_A(p)T_A(A, B, C), \quad \ldots \quad (A.1)$$

$$B = G_B(p)T_B(A, B, C), \quad \ldots \quad (A.2)$$

$$C = G_C(p)T_C(A, B, C), \quad \ldots \quad (A.3)$$

where the symbols have their usual significance as mentioned in the text. Now we have,

$$T_A(A, B, C) = [X_{out}]_A, \quad \ldots \quad (A.4)$$

$$\frac{1}{G_A(p)} = \frac{1}{GA_0}\left[1+\frac{p+\omega_{0A}{}^2 p}{\alpha_A}\right] \quad \ldots \quad (A.5)$$

$$\psi_A+\psi_C = 2\psi_B+\phi, \quad \ldots \quad (A.6)$$

where ω_{0A} is the resonant angular frequency of the tuned cricuits sustaining the mode A, G_{A0}, is the gain at the resonant frequency and ψ_A, ψ_B and ψ_C are respectively the instantaneous phases of the modes A, B and C putting $p = j\omega_A+S$ one can write from (A-5)

$$\frac{1}{G_A(p)} \simeq \left[1+\frac{2}{\alpha_A}S+j\,\frac{\omega_A{}^2-\omega_{0A}{}^2}{\alpha_A\omega_A}\right], \quad \ldots \quad (A.7)$$

where S represents an operator in a slow time scale and α_A is given by

$$\alpha_A = \omega_{0A}/Q_A \quad \ldots \quad (A.8)$$

Hence from Eqs. (A.1) and (A.7) we have

$$\frac{2}{\alpha_A}\frac{dA}{dt} \simeq G_{A0}\left[a_1A-\frac{3}{4}\,a_3\left\{A^3+2(B^2+C^2)A+B^2C\cos\phi\right\}\right]-A, \quad \ldots \quad (A.9)$$

and

$$\frac{2}{\alpha_A}\frac{d\psi_A}{dt} \simeq -\Delta\omega_A+\frac{3}{4}a_3G_{A0}B^2\frac{C}{A}\sin\phi. \quad \ldots \quad (A.10)$$

Similarly for other modes one can easily write the following equations

$$\frac{2}{\alpha_B}\cdot\frac{dB}{dt}\simeq G_{B0}\left[a_1B-\frac{3}{4}a_3\{B^3+2(A^2+C^2)B+2ABC\cos\phi\}\right]-B, \quad \dots \quad \text{(A.11)}$$

$$\frac{2}{\alpha_C}\cdot\frac{dC}{dt}\simeq G_{C0}\left[a_1C-\frac{3}{4}a_3\{C^3+2(A^2+B^2)C+B^2A\cos\phi\}\right]-C, \quad \dots \quad \text{(A.12)}$$

and

$$\frac{2}{\alpha_B}\cdot\frac{d\psi_B}{dt}\simeq\Delta\omega_B-\frac{3}{4}a_3G_{B0}AC\sin\phi, \quad \text{(A.13)}$$

$$\frac{2}{\alpha_C}\cdot\frac{d\psi_C}{dt}\simeq\Delta\omega_C+\frac{3}{4}a_3G_{C0}B^2\frac{A}{C}\sin\phi. \quad \text{(A.14)}$$

Comparing (A.10), (A.13) and (A.14) with (A.[illegible]) one can write

$$\frac{d\phi}{dt}\simeq 2\Delta\omega_B-(\Delta\omega_A+\Delta\omega_C)+K\sin\phi, \quad \text{(A.15)}$$

where

$$K=\frac{3}{8}a_3\left[B^2\left(\alpha_AG_{A0}\frac{C}{A}+\alpha_CG_{C0}\frac{A}{C}\right)-2\alpha_BG_{B0}AC\right]. \quad \dots \quad \text{(A.16)}$$

B.I. *Derivation of the loop equations with the External input* :

Let us consider the loop with the external input $E'\cos\omega t$. Assuming the instantaneous phases of the free-running modes and external input to be as

$$\psi_A+\psi_C=2\psi_B+\phi \quad \dots \quad \text{(B.1)}$$

and

$$\psi=\psi_B+\theta \quad \dots \quad \text{(B.2)}$$

where ψ is the instantaneous phase of the external signal and θ is the phase difference between the mode B and the external input. The loop equations can be written as

$$T_A(A,B,C,E)\,G_A(p)=A, \quad \dots \quad \text{(B.3)}$$

$$T_B(A,B,C,E)+E\cos\theta=\frac{B}{G_B(p)}, \quad \text{(B.4)}$$

$$T_C(A,B,C,E)\,G_C(p)=C, \quad \dots \quad \text{(B.5)}$$

where E is the amplitude of the external at the input to the non-linearity and the other symbols have their usual significances. From appendix A one can write the following expression for $G_A(P)$, $G_B(P)$ or $G_C(P)$

$$G_Z(p)\quad\left[1+\frac{2}{\alpha_Z}S+j\,\frac{\omega_Z{}^2-\omega_{0Z}{}^2}{\alpha_Z\,\omega}\right], \quad \dots \quad \text{(B.6)}$$

where $Z=A$, B or C.

Hence from equations (B.1) to (B.6) one can write

$$\frac{2}{\alpha_A}\cdot\frac{dA}{dt}\simeq G_{A0}\Big[a_1A-\frac{3}{4}a_3\{A^3+2(B^2+C^2+E^2)A+B^2C\cos\phi\}\Big]-A, \quad \text{(B.7)}$$

$$\frac{2}{\alpha_B}\cdot\frac{dB}{dt}\simeq G_{B0}\Big[a_1B-\frac{3}{4}a_3\{B^3+2(A^2+C^2+E^2)B+2ABC\cos\phi\}\Big]-B, \quad \text{(B.8)}$$

$$\frac{2}{\alpha_C}\cdot\frac{dC}{dt}\simeq G_{C0}\Big[a_1C-\frac{3}{4}a_3\{C^3+2(A^2+B^2+E^2)C+B^2A\cos\phi\}\Big]-C, \quad \text{(B.9)}$$

$$\frac{d\phi}{dt}\quad 2\Delta\omega_B-(\Delta\omega_A+\Delta\omega_C)+K\sin\phi, \quad \ldots\ \text{(B.10)}$$

and

$$\frac{d\theta}{dt}\simeq\Omega-\frac{E}{B}\cdot\frac{\omega_{0B}}{2Q_B}\cdot\sin\theta, \quad \ldots\ \text{(B.11)}$$

where θ is the instantaneous phase difference between the free-running mode and the external input and $\Omega/2\pi$ is instantaneous difference of frequency between them. Putting

$$\frac{4}{3}\cdot\frac{a_1G_{Z0}-1}{a_3G_{Z0}}=K_{Z0} \quad \ldots\ \text{(B.12)}$$

where $Z=A$, B or C.

We have from Eqs. (B.7), (B.8) and (B.9)

$$\frac{2}{\alpha_A}\cdot\frac{dA}{dt}\simeq\frac{3}{4}a_3G_{A0}A\Big[K_{A0}-\Big\{A^2+2(B^2+C^2+E^2)+B^2\frac{C}{A}\cdot\cos\phi\Big\}\Big], \ldots\ \text{(B.13)}$$

$$\frac{2}{\alpha_B}\cdot\frac{dB}{dt}\simeq\frac{3}{4}a_3G_{B0}B[K_{B0}-\{B^2+2(A^2+C^2+E^2)+2AC\cos\phi\}], \quad \ldots\ \text{(B.14)}$$

$$\frac{2}{\alpha_C}\cdot\frac{dC}{dt}\simeq\frac{3}{4}a_3G_{C0}C\Big[K_{C0}-\Big\{C^2+2(A^2+B^2+E^2)+B^2\frac{A}{C}\cos\phi\Big\}\Big]. \ldots\ \text{(B.15)}$$

REFERENCES

Chakrabarti, N. B., "Generation of pulse-like functions by means of lumped equivalent of delay lines" 1961-62, Research Report, Vol. XIV, pp. 9-. (Institute of Radio Physics and Electronics).

Chakrabarti, N. B. and Biswas, B. N., 1964, *Ind. Jour. Phys.*, **38**, 148–173,

Dishman, M. I., 1958. *Proc. I.R.E.*, **46**, 895-.

Fredendall, G. L., 1954., *Proc. I.R.E.*, pp. 258.

Met, V., 1957. *Proc. I.R.E.*, **45**, 1119.